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# Stabilization of the long-range magnetic ordering by dipolar and magnetoelastic interactions in two-dimensional ferromagnets

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**Abstract.** The possibility of the long-range magnetic order stabilization in two-dimensional ferromagnets with the account of dipolar and magnetoelastic interactions is investigated. The mechanisms of the magnetic order stabilization by both types of interactions are studied. The Curie temperature is estimated. The comparisons with experimental data are made.

**PACS.** 75.10.-b General theory and models of magnetic ordering – 75.30.Kz Magnetic phase boundaries (including magnetic transitions, metamagnetism, etc.)

## 1 Introduction

The physical properties of two-dimensional magnets essentially differ from the properties of three-dimensional magnetic systems [1]. First of all, there is no long-range magnetic ordering in two-dimensional isotropic magnets and easy-plane ones [2–4]. In [5] it was shown, that the interaction of Holdstone particles in a two-dimensional ferromagnet leads to the "infrared" divergence. Besides in such systems the existence of the condensate of superconducting electron pairs is impossible [6]. However, it is experimentally established that there is the long-range magnetic ordering at finite temperature in clean monolayers [7].

Easy-axis anisotropy breaks the symmetry of the system, resulting in the gap in magnon spectrum and stabilities the long-range ferromagnetic order. In real magnets the dipolar interaction is present and the account of this interaction leads to stabilization of the long-range magnetic order. As it is known, the energy of magnons of an isotropic two-dimensional ferromagnet (with the account of just the exchange interaction) is proportional to $k^2(\omega(k) \propto k^2)$, where $k$ is the wave vector. The fluctuation integral diverges on the lower limit, that testifies to the absence of the long-range magnetic order in the system. The account of dipolar interaction leads to the square-root modification of the spin-wave dispersion relation: $\omega(k) \propto \sqrt{k}$, and the fluctuation integral converges on the lower limit [8].

The account of both single-ion anisotropy (easy-axis and easy-plane one) and dipolar interaction was carried out in very interesting paper [10]. In such a system there is the long-range magnetic order due to the dipolar interaction and the easy-axis anisotropy. The similar analysis was carried out for various two-dimensional ferromagnets [11,12].

In real magnetic systems both this interactions play an essential role. However, in such systems there also exists the magnetoelastic (ME) coupling. It is known, that this interaction (in spite of the fact that it is weak in magnets) changes the spin-wave dispersion relation, resulting in the magnetoelastic gap in quasimagnon spectrum. The effect of ME coupling is essential near the point of orientational phase transitions. It stabilizes the long-range magnetic order in two-dimensional magnets [9,13]. So it is of interest to investigate the properties of two-dimensional ferromagnet with the account of ME and dipolar interactions. The aim of our paper is to study the simultaneous influence of this interactions on spin-wave spectra and stabilization of long-range order.

The Hamiltonian of the ferromagnet is as follows:

$$\begin{aligned}\mathsf{H} = &-\frac{1}{2}\sum_{n,n'}\sum_{\alpha,\beta}\left(J_{nn'}\delta_{\alpha\beta} + V_{nn'}^{\alpha\beta}\right)S_n^\alpha S_{n'}^\beta \\ &+ \lambda\sum_n\left\{\left(S_n^x\right)^2 u_{xx}\left(S_n^z\right)^2 u_{zz} + \left(S_n^x S_n^z + S_n^z S_n^x\right)u_{xz}\right\} \\ &+ \int \mathrm{d}\nu \frac{E}{2(1-\sigma^2)}\left\{u_{xx}^2 + u_{zz}^2 + 2\sigma u_{xx}u_{zz} + 2(1-\sigma)u_{xz}^2\right\}\end{aligned} \tag{1}$$

where $J_{nn'}$ is the exchange integral,

$$V_{nn'}^{\alpha\beta} = (g\mu)^2\left(3R_{nn'}^\alpha R_{nn'}^\beta - \delta_{\alpha\beta}R_{nn'}^2\right)R_{nn'}^{-5}, \tag{2}$$

- is the tensor of dipolar interaction, $g$ stands for $g$-factor, $\mu$ is Bohr's magneton, $\mathbf{R}_{nn'}$ is the radius-vector of sites $n$ and $n'$, $S_n^i$ is the $i$th component of the spin operator at the lattice site $n$, $\lambda$ is the magnetoelastic coupling constant, $\mu_{ij}$ is the symmetric part of deformation tensor;

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$E$ is Young's modulus, $\sigma$ is the Poisson coefficient. The plane of the film coincides with the $X0Z$ plane.

The first term in (1) describes the exchange and dipolar coupling, the second is the magnetoelastic coupling, the third term includes the energy of elastic deformations of the lattice, which we assume to be isotropic [14].

To simplify the calculations we assume, that the spin of a magnetic ion $S$ is equal to unity. We also suppose that there exists a nonzero magnetic moment aligned with the $0Z$ axis.

In the further calculations we use the known values of a dipolar tensor [8]:

$$\begin{aligned} V_k^{xx} &= \frac{A_0}{3} - k\Omega_0 \sin^2\varphi_k,\ V_k^{yy} = -\frac{2A_0}{3} + k\Omega_0, \\ V_k^{zz} &= \frac{A_0}{3} - k\Omega_0 \cos^2\varphi_k,\ V_k^{xz} = -k\Omega_0 \frac{\sin 2\varphi_k}{2}, \\ A_0 &= \frac{3}{2}(g\mu)^2 \sum_{R\neq 0} R^{-3},\quad \Omega_0 = \frac{2\pi(g\mu)^2}{\nu_2}, \end{aligned} \tag{3}$$

where $\nu_2$ is the "volume" of a flat unit cell, $\varphi_k$ is the angle between the direction of the wave vector and magnetization vector.

## 2 Dispersion relation of coupled magnetoelastic waves

We will use the Hubbard operators technique and bosonization method which enable us to take into account the magnetoelastic coupling exactly, and exchange and dipole interactions in the molecular field approximation [13,15]. These methods have substantial advances in comparison with the Holstein-Primakoff transformation. We investigate the system with finite value of magnetic ion spin ($S = 1$), while the Holstein-Primakoff transformation is just expansion on $1/S$, and it is valid for very large $S$ [22].

Because $V_0^{xz} = 0$, after separation of the mean field from (1), we obtain the one-site Hamiltonian:

$$\mathsf{H}_0(n) = -J_z S_n^z + \lambda \left\lfloor u_{xx}(S_n^x)^2 + u_{zz}(S_n^z)^2 \right\rfloor, \tag{4}$$

where $J_z = \langle S\rangle(J_0 + V_0^{zz})$.

Taking into account the action of $S^{\pm}$, $S^z$ operators on $S^z$ eigenvectors:

$$\begin{aligned} S_n^z|n;M\rangle &= M|n;M\rangle;\ S_n^+|n;M\rangle \\ &= \sqrt{(S+1)\cdot(S+M+1)}|n;M+1\rangle; S_n^- \\ &= \left(S_n^+\right)^+, \end{aligned}$$

one can expand the eigenvectors of $\mathsf{H}_0$ in this basis. Solving the equation with the Hamiltonian (4): $\mathsf{H}_0(n)\Psi_n(M) = E_M\Psi_n(M)$, where $-S \le M \le S$, we find the energy levels of a magnetic ion $E_M$ and eigenfunctions of the Hamiltonian $\mathsf{H}_0$:

$$\begin{aligned} E_1 &= \frac{\lambda}{2}\left(u_{xx}^{(0)} + 2u_{zz}^{(0)}\right) - \chi,\quad E_0 = \lambda u_{xx}^{(0)}, \\ E_{-1} &= \frac{\lambda}{2}\left(u_{xx}^{(0)} + 2u_{zz}^{(0)}\right) + \chi,\quad \chi^2 = J_z^2 + \left(\frac{\lambda}{2}u_{xx}^{(0)}\right)^2. \end{aligned} \tag{5}$$

$$\begin{aligned} \Psi(1) &= \cos\delta|1\rangle + \sin\delta|-1\rangle,\quad \Psi(0) = |0\rangle, \\ \Psi(-1) &= -\sin\delta|1\rangle + \cos\delta|-1\rangle\cdot \end{aligned} \tag{6}$$

Here

$$\cos\delta = \frac{\lambda u_{xx}^{(0)}}{2\sqrt{(\chi - J_z)^2 + \left(\frac{\lambda u_{xx}^{(0)}}{2}\right)^2}},$$

$u_{ij}^{(0)}$ are the spontaneous deformations, obtained from the condition of the free-energy density minimum ($F = F_{el} - T\ln Z$, where $Z$ is the partition function). The nonzero spontaneous deformations are:

$$u_{xx}^{(0)} = -\frac{\lambda}{E}\frac{1-2\sigma}{2},\ u_{zz}^{(0)} = -\frac{\lambda}{E}\frac{2-\sigma}{2},\ u_{xz}^{(0)} = 0. \tag{7}$$

The Hubbard operators are built on the basis of eigenfunctions (6) [15,16]:

$$X_n^{M'M} \equiv |\Psi_n(M')\rangle\langle\Psi_n(M)|, \tag{8}$$

and describe the transition of a magnetic ion from the state with a quantum number $M'$ to the state with a quantum number $M$. The relation of spin operators with Hubbard operators reads:

$$\begin{aligned} S_n^+ &= \sum_M \Gamma_\perp(M)H_n^M + \sum_\alpha \gamma_\perp(\alpha)X_n^\alpha, \\ S_n^- &= \sum_M \Gamma_\perp^*(M)H_n^M + \sum_\alpha \gamma_\perp^*(-\alpha)X_n^\alpha, \\ S_n^z &= \sum_M \Gamma_\parallel(M)H_n^M + \sum_\alpha \gamma_\parallel(\alpha)X_n^\alpha. \end{aligned} \tag{9}$$

Here $H_n^M \equiv X_n^{MM}$ are diagonal Hubbard operators, $\alpha$ are the root vectors, which algebra is defined by such commutation relations [15]:

$$\left\lfloor H_n^M, X_n^{pq}\right\rfloor = (\delta_{Mp} - \delta_{qM})X_n^{pq} = \alpha(p,q)X_n^{\alpha(p,q)}.$$

For the system with spin $S$ each root vector $\alpha$ has $2S+1$ components.

In our case the eigenvectors has the form (9) so this relation looks like:

$$\begin{aligned} S_n^+ &= \sqrt{2}\cos\delta\left(X_n^{10} + X_n^{0-1}\right) + \sqrt{2}\sin\delta\left(X_n^{01} - X_n^{-10}\right), \\ S_n^- &= \sqrt{2}\cos\delta\left(X_n^{01} + X_n^{-10}\right) + \sqrt{2}\sin\delta\left(X_n^{10} - X_n^{0-1}\right), \\ S_n^z &= \cos 2\delta\left(H_n^1 - H_n^{-1}\right) - \sin 2\delta\left(X_n^{1-1} - X_n^{-11}\right). \end{aligned} \tag{10}$$

Comparing (9) with (10), for $\gamma_{\|}(\alpha)$, $\gamma_{\perp}(\alpha)$ we obtain:

$$\gamma_{\|}(1-1) = \gamma_{\|}(-11) = -\sin 2\delta,$$
$$\gamma_{\perp}(10) = \gamma_{\perp}(0-1) = \sqrt{2}\cos\delta,$$
$$\gamma_{\perp}(0-1) = -\gamma_{\perp}(-10) = \sqrt{2}\sin\delta. \qquad (11)$$

The further analysis we shall carry out in the mean field approximation, therefore we need only the "transverse" part of the Hamiltonian, which we can obtain, using the relation between spin operators and Hubbard operators (9, 10):

$$\mathsf{H}_{int}^{\perp} = -\frac{1}{2}\sum_{nn'}\sum_{\alpha\beta} X_n^{\alpha}\,\mathbf{C}(\alpha)A_{nn'}\,\mathbf{C}^T(\beta)X_{n'}^{\beta}, \qquad (12)$$

where $X_n^{\alpha}$ is the Hubbard operator, the vector $\mathbf{C}(\alpha)$ and the matrix $A_{nn'}$ look like:

$$\mathbf{C}(\alpha) = (\gamma_{\|}(\alpha) \quad \gamma_{\perp}(\alpha) \quad \gamma_{\perp}^{*}(-\alpha)),$$

$$A_{nn'} = \begin{pmatrix} J_{nn'} + V_{nn'}^{zz} & \frac{V_{nn'}^{xz}}{2} & \frac{V_{nn'}^{xz}}{2} \\ \frac{V_{nn'}^{xz}}{2} & \frac{V_{nn'}^{xx} - V_{nn'}^{yy}}{4} & \frac{J_{nn'}}{2} + \frac{V_{nn'}^{xx} + V_{nn'}^{yy}}{4} \\ \frac{V_{nn'}^{xz}}{2} & \frac{J_{nn'}}{2} + \frac{V_{nn'}^{xx} + V_{nn'}^{yy}}{4} & \frac{V_{nn'}^{xx} - V_{nn'}^{yy}}{4} \end{pmatrix}. \qquad (13)$$

The one-site Hamiltonian $\mathsf{H}_0$, being recast in the terms of Hubbard operators becomes diagonal one:

$$\mathsf{H}_0 = \sum_n (\mathbf{E}\mathbf{H}_n).$$

Represent the components of the deformation tensor in the form $u_{ij} = u_{ij}^{(0)} + u_{ij}^{(0)}$, where $u_{ij}^{(0)}$ are spontaneous deformations of the lattice, $u_{ij}^{(1)}$ is the dynamic part of the deformation tensor, describing the oscillations of the lattice ions. The deformations $u_{ij}^{(1)}$ can be presented through operators of creation $b_{k,\mu}^{+}$ and annihilation $b_{k,\mu}$ of phonons [17]:

$$u_{ij}^{(1)} = \frac{\mathrm{i}}{2}\sum_{k,\mu}\frac{\exp(\mathrm{i}\mathbf{k}\mathbf{n})}{\sqrt{2mN\omega_{\mu}(k)}}\left(b_{k,\mu} + b_{-k,\mu}^{+}\right)\left(e_{\mu}^{j}k_i + e_{\mu}^{i}k_j\right),$$

where $\mathbf{k}$ is the wave vector, $m$ is the atom mass, $N$ is the number of sites in the crystalline lattice, $\omega_{\mu}(k) = c_{\mu}k$ is the dispersion relation for a $\mu$-polarized phonon and $c_{\mu}$ is the velocity of sound, $\mathbf{e}_{\mu}(k)$ is the unit vector of phonon polarization.

Separating from the one-site Hamiltonian the terms proportional to $u_{ij}^{(1)}$, and using the above mentioned formula, we obtain the Hamiltonian, which describes the transformation of magnons into phonons and *vice versa*:

$$\mathsf{H}_{tr} = \sum_n \left\{\sum_M P_M H_n^M + \sum_{\alpha} P_{\alpha} X_n^{\alpha}\right\}, \qquad (14)$$

where:

$$P_{M(\alpha)} = \frac{1}{\sqrt{N}}\sum_{k,\mu}\left(b_{k,\mu} + b_{-k,\mu}^{+}\right)T_n^{M(\alpha)}(k,\mu),$$

$T_n^{M(\alpha)}(k,\mu)$ are the transformation amplitudes.

For the system under investigation the transformation amplitudes look like:

$$T_n^{11}(k,\mu) = \lambda T_n^0(k,\mu)\left(e_{\mu}^{z}k_z + \frac{1+\sin 2\delta}{2}e_{\mu}^{x}k_x\right),$$
$$T_n^{00}(k,\mu) = \lambda T_n^0(k,\mu)e_{\mu}^{x}k_x,$$
$$T_n^{-1-1}(k,\mu) = \lambda T_n^0(k,\mu)\left(e_{\mu}^{z}k_z + \frac{1-\sin 2\delta}{2}e_{\mu}^{x}k_x\right),$$
$$T_n^{1-1}(k,\mu) = T_n^{-11}(k,\mu) = \frac{1}{2}\lambda T_n^0(k,\mu)\cos 2\delta(e_{\mu}^{x}k_x),$$
$$T_n^{01}(k,\mu) = T_n^{10}(k,\mu) = \frac{\lambda}{4}T_n^0(k,\mu)(e_{\mu}^{z}k_x + e_{\lambda}^{x}k_z)(\cos\delta - \sin\delta),$$
$$T_n^{0-1}(k,\mu) = T_n^{-10}(k,\mu) = -\frac{\lambda}{4}T_n^0(k,\mu)(e_{\mu}^{z}k_x + e_{\mu}^{x}k_z)(\cos\delta + \sin\delta), \qquad (15)$$

The Hamiltonian (1) can be presented in the form:

$$\mathsf{H} = \mathsf{H}_{int}^{\perp} + \mathsf{H}_{tr} + \mathsf{H}_0, \qquad (16)$$

where $\mathsf{H}_0$ is the diagonalized one-site Hamiltonian, $\mathsf{H}_{int}^{\perp}$ is the transverse part of $\mathsf{H}_{int}$ (12), $\mathsf{H}_{tr}$ is the Hamiltonian of transformations (14).

As it is well established [18,19], the spectra of elementary excitations are determined by the poles of the Green function, which we define as follows:

$$G^{\alpha\alpha'}(n,\tau;n',\tau') = -\left\langle \hat{T}X_n^{\alpha}(\tau)X_{n'}^{\alpha'}(\tau')\right\rangle. \qquad (17)$$

Here $X_n^{\alpha}(\tau) = \mathrm{e}^{H\tau}X_n^{\alpha}\mathrm{e}^{-H\tau}$ are the Hubbard operators in the Heisenberg representation, $\hat{T}$ is Wick's operator. The averaging is carried out with the Hamiltonian (16).

$$\det\left\|\delta_{ij} + G_0^{\alpha} b(\alpha) a_{ip}(\alpha) A_{pj} + B^0(k;\mu,\mu') T^{-\alpha}_{k,\mu} G_0^{\alpha} b(\alpha) T^{\beta}_{-k,\mu'} G_0^{\beta} b(\beta) a_{ip}(\alpha,\beta) A_{pj}\right\| = 0 \tag{18}$$

In our case the dispersion equation of hybridized magnetoelastic waves obtained from the solution of Larkin's equation [18–20], with the Hamiltonian (1), has the form:

*See equation (18) above*

where $B^0(k,\lambda,\lambda') = \frac{D_\lambda(k,\omega_n)}{1-Q_{\lambda\lambda'}D_\lambda(k,\omega_n)}$; $Q_{\lambda\lambda'} = T^{\alpha}(-k,\lambda)G_0^{\alpha}(\omega_n)T^{-\alpha}(k,\lambda') = a_{ik}\alpha,\beta) = (\mathbf{C}^T(\alpha)\mathbf{C}(-\beta))_{ik}$, $a_{ik}(\alpha) = a_{ik}(\alpha,\alpha), b(\alpha) = \langle\boldsymbol{\alpha}\mathbf{H}\rangle_0$, $D_\lambda(k,\omega_n) = \frac{2\omega_\lambda(k)}{\omega_n^2-\omega_\lambda^2(k)}$ is the Green function of free $\lambda$-polarized phonon, $G_0^{\alpha}(\omega) = \{\omega + (\boldsymbol{\alpha}\mathbf{E})\}^{-1}$ is zero Green function.

The solution of the dispersion equation (18) determines the spectra of magnon excitations of the ferromagnet at arbitrary values of temperature, spin or singe-ion anisotropy constant.

Using the fact that $\sin\delta \ll 1$, from (18) we obtain the expression for the quasimagnon spectra in the case of low temperatures:

$$\varepsilon^2(k) = \left(A_0 + b_0 + \alpha k^2\right)\left(b_0 + k\Omega_0 \sin^2\varphi_k + \alpha k^2\right), \tag{19}$$

where $\alpha = J_0 R_0^2$, $R_0$ is the radius of interaction, $b_0 = \frac{3\lambda^2}{4E}$.

It is evident from (19) that in the case $\lambda = 0$ we obtain Maleev's result [8]:

$$\varepsilon(k) = |\sin\varphi_k|\sqrt{kA_0\Omega_0}. \tag{20}$$

Without the dipolar interaction ($A_0 = 0$, $\Omega_0 = 0$) we obtain a quasimagnon spectrum, similar to that obtained in [20]:

$$\varepsilon^2(k) = \{b_0 + \chi - J_k(1-\sin 2\delta)\}\{b_0 + \chi - J_k(1+\sin 2\delta)\},$$

which at $\delta \to 0$ (we take, that $\langle S^z\rangle \approx 1$) can be presented as:

$$\varepsilon(k) = b_0 + \alpha k^2. \tag{21}$$

## 3 Fluctuation integral and Curie temperature

Let us consider the fluctuation of the magnetic moment and estimate the probability of the existence of the long-range magnetic ordering. The simplest way to do this is to represent the Hubbard operators through bose operators using the bosonization method [21]. It is necessary to note that further calculations we shall carry out taking $\mathbf{k} \parallel OX$, *i.e.* the angle $\varphi_k = \pi/2$.

The method is as follows. The one-site Hamiltonian should be diagonalized, and the Hamiltonian can be recast in terms of Hubbard operators. We assign pseudohubbard operators $\tilde{X}_n^{\alpha}$ to Hubbard operators $X_n^{\alpha}$, and connect them with Bose operators by relations:

$$\begin{gathered}\tilde{X}_n^{10} = (1 - a_n^+ a_n - b_n^+ b_n)a_n,\ \tilde{X}_n^{01} = a_n^+,\\ \tilde{X}_n^{1-1} = (1 - a_n^+ a_n - b_n^+ b_n)b_n,\\ \tilde{X}_n^{-11} = b_n^+,\ \tilde{X}_n^{0-1} = a_n^+ b_n,\ \tilde{X}_n^{-10} = b_n^+ a_n,\ \tilde{H}_n^0 = a_n^+ a_n,\\ \tilde{H}_n^{-1} = b_n^+ b_n,\ \tilde{H}_n^1 = 1 - a_n^+ a_n - b_n^+ b_n.\end{gathered} \tag{22}$$

The fluctuations of the magnetic moment are determined by the integral:

$$\langle \Delta M\rangle \propto \int_0^{\infty} N(\varepsilon) k \mathrm{d}k,$$

where

$$N(\varepsilon) = \left(\mathrm{e}^{\frac{\varepsilon(k)}{T}} - 1\right)^{-1}.$$

The convergence of this integral on the lower limit testifies to the stabilization of the long-range magnetic ordering.

We decide on the possibility of the long-range magnetic ordering from the convergence of:

$$\langle S^z\rangle = \left\langle H^1 - H^{-1}\right\rangle. \tag{23}$$

Substituting (10) into (23), we obtain:

$$\langle S_n^z\rangle = 1 - 2\left\langle b_n^+ b_n\right\rangle - \left\langle a_n^+ a_n\right\rangle. \tag{24}$$

The Hamiltonian (1) can be presented in the form:

$$\begin{aligned}\mathsf{H}^{(2)} = \sum_k \Bigg(&E_0 - E_1 - J_k - \frac{1}{2}(1+\sin 2\delta)V_k^{xx}\\ &-\frac{1}{2}(1-\sin 2\delta)V_k^{yy}\Bigg)\cdot a_k^+ a_k\\ &+\frac{1}{2}\sum_k\Bigg(-J_k\sin 2\delta - \frac{1}{2}(1+\sin 2\delta)V_k^{xx}\\ &+\frac{1}{2}(1-\sin 2\delta)V_k^{yy}\Bigg)\left(a_k a_{-k} + a_k^+ a_{-k}^+\right)\\ &+\sum_k (E_{-1}+E_1)\cdot b_k^+ b_k.\end{aligned}$$

Using the $u-\nu$ transformation [22],

$$\begin{aligned}a_k^+ &= u_k\alpha_k + \nu_k^*\alpha_{-k}^+,\\ a_k &= u_k^*\alpha_k^+ + \nu_k\alpha_{-k},\end{aligned} \tag{25}$$

it is easy to obtain the boson energies:

$$\mathsf{H}^{(2)} = \sum_k \varepsilon_\alpha \alpha_k^+\alpha_k + \sum_k \varepsilon_b b_k^+ b_k,$$

where $\varepsilon_\alpha$ is determined by (19), and $\varepsilon_b$ is the high-frequency quasimagnon branch:

$$\varepsilon_b = 2\chi.$$

Since $\sqrt{b_0(A_0+b_0)} \ll 2\chi$, in further calculations we do not take into account the high-frequency quasimagnon branch.

Substituting expressions (25) to the formula (24), and using the fact that:

$$\frac{1}{N}\sum_n \langle \alpha_n^+ \alpha_n \rangle = \frac{1}{(2\pi)}\int_0^\infty \frac{k\mathrm{d}k}{\mathrm{e}^{\varepsilon_\alpha/T}-1},$$

we obtain the result:

$$\frac{1}{N}\sum_n \langle S^z \rangle = 1 - \frac{1}{(2\pi)}\int_0^\infty \frac{\left(|u_k|^2+|\nu_k|^2\right)k\mathrm{d}k}{\mathrm{e}^{\varepsilon_\alpha/T}-1}\cdot \quad (26)$$

Here $|u_k|^2+|\nu_k|^2 \propto \frac{1}{\varepsilon_\alpha}$. It is evident, that if $\varepsilon_\alpha$ is determined by the expression (19), the integral (26) converges on the lower limit, that testifies to the existence of the long-range magnetic ordering in the system.

Such convergence of the integral is caused by the presence of the magnetoelastic gap $b_0$ in quasimagnon spectrum. In the absence of magnetoelastic coupling the integral also converges, but this convergence is stipulated by a square-root dependence of quasimagnon spectrum on the wave vector $\mathbf{k}$ in (20).

## 4 Conclusion

Thus, the mechanisms of the long-range magnetic ordering stabilization by magnetoelastic and dipolar interactions are different. In the first case the long-range magnetic ordering is stabilized due to magnetoelastic gap in the spectrum, while the dipolar interaction changes the dependence of spectrum on the wave vector. The simultaneous account of both magnetoelastic and dipolar interaction leads to combining of these effects: the spectrum still has square root dependence on the wave vector $\mathbf{k}$ but possesses the ME gap $b_0$.

From the requirement

$$\frac{1}{N}\sum_n \langle S^z \rangle = 0,$$

one can estimate the Curie temperature. This equation yields:

$$\frac{1}{(2\pi)}\int_0^\infty \frac{\left(|u_k|^2+|\nu_k|^2\right)k\mathrm{d}k}{\mathrm{e}^{\varepsilon_\alpha/T_C}-1} = 1. \quad (27)$$

Let us consider the solutions of (27) in the two cases: $\lambda \to 0$ and $A_0, \Omega_0 \to 0$. In the first case (just dipolar interaction), we obtain the well known Maleev's result [8]:

$$T_C \approx \frac{\alpha}{\ln\left(\dfrac{\alpha}{\Omega_0}\right)}.$$

If we account only the magnetoelastic interaction the solution of (27) gives the following result:

$$T_C \approx \frac{4\pi\alpha}{\ln\left(\dfrac{4\pi\alpha}{b_0}\right)}\cdot \quad (28)$$

Without magnetoelastic coupling ($b_0 \to 0$), from (28) we obtain $T_C \to 0$, that is in agreement with the Mermin-Wagner theorem [2].

The simultaneous account of magnetoelastic and dipolar interactions essentially complicates the integral in the l.h.s. of (27). However, one can estimate the asymptotic of this integral on the lower limit, that enables us to calculate the Curie temperature:

$$T_{C3} \approx 4\pi\alpha\left\{\ln\sqrt{\frac{\alpha(A_0+2b_0)}{b_0(A_0+b_0)}} - D\mathrm{arctg}D\right\}^{-1}, \quad (29)$$

where

$$D = \frac{\Omega_0}{\sqrt{4\dfrac{\alpha}{b_0}(A_0+b_0)(A_0+b_0)-\Omega_0^2}}.$$

If magnetoelastic parameter $b_0$ is much less then dipolar parameter, *i.e.* $b_0 \ll A_0$ the expression (28) has more simple form:

$$T_{C3} \approx 4\pi\alpha\left\{\ln\sqrt{\frac{\alpha}{b_0}} - \frac{\Omega_0}{\sqrt{4\dfrac{\alpha}{b_0}A_0^2-\Omega_0^2}}\mathrm{arctg}\frac{\Omega_0}{\sqrt{4\dfrac{\alpha}{b_0}A_0^2-\Omega_0^2}}\right\}^{-1}, \quad (30)$$

As well as the spectra of quasimagnons, the Curie temperature depends on the magnetoelastic and dipolar parameters in different ways.

One can estimate the Curie temperature using the obtained results. First of all one should note that these formulae give us rough estimates of $T_C$ because we use the condition $S = 1$, while in experiments such films as Fe/Ag, Co/Ag etc. with $S > 1$ are investigated. Besides we have not taken into account the high temperature effects.

Despite all this, from the exact integral (27), we obtain in the case of the absence of ME coupling ($\lambda \to 0$): $T_{C,d} \approx 540$ K for values of Co fcc(001), Fe/Ag(001) films [10] $H_{ex} \approx 2000$ kOe, $\Omega_0 \approx 14$ kOe. In the case of the absence of dipolar interaction ($A_0, \Omega_0 \to 0$) we have the much less value $T_{C,ME} \approx 290$ K where we take $b_0 \approx 2.7$ Oe [24]. The simultaneous account of both these interactions increases the critical temperature up to the value of $570 \sim 590$ K.

Here we can compare our results with experiments values of $T_C$. For films Fe/Ag(001) of 1 ML-thick $T_C \approx 400$ K [23]. For Co fcc(001) films the $T_C$ is about 500 K ([10], and therein).

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